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MSW effect with flavor-changing neutrino interactions

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We consider the effect that flavor non-diagonal neutrino interactions with matter have on the resonant ν oscillations. It is shown that, even in the absence of ν mixing in vacuum, an efficient conversion of the electron neutrinos from the sun to another ν flavor can result if the strength of this interaction is $\sim 10^{-2} G_F$. We show how this can be implemented in the minimal supersymmetric standard model with R-parity breaking. Here, the L violating couplings induce neutrino masses, mixings and the flavor non-diagonal neutrino interactions that can provide an MSW-like solution to the solar neutrino problem even for negligible vacuum mixings.



I— The most elegant solution to the solar neutrino problem is based in the so-called MSW effect¹⁻⁴, i.e. the resonant oscillation of ν_e into ν_μ or ν_τ induced by the neutrino interactions with the medium in the sun. The effect of the matter in the neutrino oscillations is related here to the fact that electron neutrinos, unlike ν_μ or ν_τ , have charged current interactions due to W exchange with the electrons in the medium. These interactions produce a potential energy for the ν_e that leads, when the electron density corresponds to the resonance value, to a maximum neutrino mixing angle in matter, even if the mixing angle in vacuum θ is small. Hence, neutrinos crossing a resonance may be subject to a significant flavor conversion.

It was already noted by Wolfenstein¹ that even in the absence of neutrino masses (and hence of mixing angles in vacuum) there could be matter induced neutrino oscillations in the presence of flavor non-diagonal neutrino interactions with the medium. Our purpose here is to study the effects that these interactions can have in the resonant neutrino conversion, and the requirements they should fulfill in order to provide a solution to the solar neutrino problem. Furthermore, we will describe how the necessary ingredients can be obtained in the minimal supersymmetric standard model with R-parity violating interactions, which can provide the required neutrino masses and interactions. Since the effects of the flavor non-diagonal ν interactions mimic those of a non-vanishing vacuum mixing angle, an MSW-like solution to the solar neutrino problem can be obtained even for vanishingly small θ .

II — Let us first quickly review the MSW effect with vacuum neutrino mixing and ordinary neutrino interactions⁵. For definiteness we will concentrate on the two generation case. The evolution for the two flavor neutrinos is given by

$$i \frac{d}{dx} \nu_c = \frac{1}{2E} M^2 \nu_c \quad (1)$$

where $\nu_c^T = (\nu_e, \nu_\alpha)$, with $\alpha = \mu$ or τ , and E is the neutrino energy. The matrix M^2 is, neglecting an overall irrelevant (as far as oscillations are concerned) phase,

$$M^2 = \frac{1}{2} \left[R_\theta \begin{pmatrix} -\Delta & 0 \\ 0 & \Delta \end{pmatrix} R_\theta^\dagger + 2E \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix} \right] \quad (2)$$

where $A = \sqrt{2} G_F N_e$ is due to the electron neutrino coherent forward scattering from electrons (of number density N_e) and $\Delta \equiv m_2^2 - m_1^2$ is the squared mass difference of the vacuum mass eigenstates (ν), that are related to the current eigenstates (ν_c) by

$$\nu_c = R_\theta \nu, \quad \text{with} \quad R_\theta = \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}. \quad (3)$$

We denote $s\theta \equiv \sin \theta$, $c\theta \equiv \cos \theta$, etc..

The matter mass eigenstates are defined by $\nu_m \equiv R_{\theta_m}^\dagger \nu_c$, with

$$R_{\theta_m}^\dagger M^2 R_{\theta_m} = \frac{1}{2} \begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}, \quad (4)$$

where $\Delta_m = \Delta \sqrt{(a - c2\theta)^2 + s^2 2\theta}$ with $a = 2EA/\Delta$. The matter mixing angle θ_m satisfies

$$s^2 2\theta_m = \frac{s^2 2\theta}{(c2\theta - a)^2 + s^2 2\theta}. \quad (5)$$

Hence, there is maximum mixing in matter in the 'resonance region' corresponding to $a = c2\theta$, and the width of this resonance corresponds to the electron density for which $|a - c2\theta| = |s2\theta|$.

The ν_m evolution is then determined by

$$i \frac{d}{dx} \begin{pmatrix} \nu_m^1 \\ \nu_m^2 \end{pmatrix} = \begin{pmatrix} -\Delta_m/4E & -i \frac{d}{dx} \theta_m \\ i \frac{d}{dx} \theta_m & \Delta_m/4E \end{pmatrix} \begin{pmatrix} \nu_m^1 \\ \nu_m^2 \end{pmatrix}, \quad (6)$$

with

$$\frac{d\theta_m}{dx} = \frac{1}{2} \frac{s2\theta}{(a - c2\theta)^2 + s^2 2\theta} \frac{da}{dx}. \quad (7)$$

Hence, in a medium with varying density, $N_e = N_e(x)$, transitions between the matter mass eigenstates are induced by a nonzero $d\theta_m/dx$. These transitions are usually negligible unless the neutrinos are near the resonance layer, for which the diagonal elements in eq. (6) are minimum and $d\theta_m/dx$ is enhanced. If $P = |\langle \nu_m^1 | \nu_m^2 \rangle|^2$ is the probability of $\nu_m^1 \rightarrow \nu_m^2$ conversion in the resonance crossing, the averaged probability to detect an electron neutrino that has crossed a resonance is

$$P_{\nu_e \nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P \right) c2\theta \times c2\theta_m, \quad (8)$$

where θ_m is the matter mixing angle corresponding to the point where the ν_e was produced, while θ is the vacuum angle. Under the assumption that the electron density varies linearly in the resonance layer, the probability of level crossing at resonance is found to be⁴

$$P = e^{-\pi\gamma/2}, \quad (9)$$

where the adiabaticity parameter γ is

$$\gamma \equiv \frac{\Delta_m}{4E |d\theta_m/dx|} = \frac{\Delta}{2E} \frac{s^2 2\theta}{c2\theta} \frac{1}{d \ln N_e / dx|_r} \quad (10)$$

(the sub-index r stands for the resonance value). In the adiabatic case, i.e. $\gamma \gg 1$, the off-diagonal terms in eq. (6) can be neglected even at resonance, so that $P = 0$ and the probability of having a ν_e after adiabatic flavor conversion is $P_{\nu_e \nu_e} = (1 + c2\theta \times c2\theta_m)/2$.

A solution to the solar neutrino deficit observed at the Davis experiment⁶ with adiabatic neutrino evolution is obtained for $\Delta \simeq 10^{-4}$ eV² (if the resonant layer is not too narrow, i.e. $s2\theta \gtrsim 10^{-2}$) and also for the so-called large angle solution ($s2\theta \sim 0.8$, 10^{-4} eV² $> \Delta > 10^{-8}$ eV²). A non-adiabatic solution exists for $\Delta \times s^2 2\theta \simeq 10^{-7.5}$ eV², closing a 'triangle' in the $\Delta - s^2 2\theta$ plane. The reduction factor in the neutrino flux depends on the ν energy in different ways in the three regimes. Hence, experiments with other energy thresholds or capable of measuring the neutrino spectra can distinguish among the three solutions. The results of the Kamiokande detector⁷ in fact disfavor the $\Delta \simeq 10^{-4}$ eV² adiabatic solution, while the preliminary observation of a very low neutrino rate at the gallium SAGE experiment, if confirmed, will support the non-adiabatic solution.

III – Let us now assume the existence of additional ν interactions leading to an effective lagrangian including flavor non-diagonal ν interactions

$$\mathcal{L}_{eff} = \frac{1}{\sqrt{2}} \bar{\nu}_j \gamma^\mu (1 - \gamma_5) \nu_i [G_{ij}^e \bar{e} \gamma_\mu e + G_{ij}^q \bar{q} \gamma_\mu q]. \quad (11)$$

We have not included other possible couplings, such as to axial vector currents, since for an unpolarized medium only the time component of the vector current leads to a significant scattering cross section in the non-relativistic limit. This leads to an additional contribution to the matrix M^2 describing the ν evolution in matter

$$\frac{M_{ij}^2}{2E} \rightarrow \frac{M_{ij}^2}{2E} + \sqrt{2} (G_{ij}^e N_e + G_{ij}^p N_p + G_{ij}^n N_n), \quad (12)$$

where $N_{n,p}$ are the neutron and proton densities and $G^n = 2G^d + G^u$, while $G^p = 2G^u + G^d$.

Clearly we expect $|G_{ij}| \ll G_F$ (see below for specific bounds), so that the effects of the diagonal elements G_{ii} in the neutrino oscillations can be neglected with respect to those of the charged current ν_e interaction. Hence, the evolution equation for the two flavor neutrinos now becomes

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \simeq \frac{\Delta}{4E} \begin{bmatrix} -c2\theta + a & s2\theta + b \\ s2\theta + b & c2\theta - a \end{bmatrix} \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} \quad (13)$$

where $b = \sqrt{2} (G_{e\alpha}^e N_e + G_{e\alpha}^p N_p + G_{e\alpha}^n N_n) 4E/\Delta$.

Thus, the mixing angle in matter is given by

$$s^2 2\theta_m = \frac{(b + s2\theta)^2}{(c2\theta - a)^2 + (b + s2\theta)^2}, \quad (14)$$

and since we expect $|db/dx| \ll |da/dx|$, the resonance width is obtained from

$$|a - c2\theta| \simeq |b_r + s2\theta|, \quad (15)$$

where b_r is the resonance value of b , i.e. corresponding to $a = c2\theta$.

Similarly, the remaining expressions can be obtained with the substitution $s2\theta \rightarrow b + s2\theta$ in the expressions (4-10) for the ordinary MSW effect. Since at resonance $N_e = \frac{c2\theta}{\sqrt{2}} \frac{\Delta}{2EG_F}$, it follows that

$$b_r = \frac{2c2\theta}{G_F} (G^e + G^p + Y_n G^n)_{e\alpha}, \quad (16)$$

with $Y_n \equiv N_n/N_e$. Note that b_r is independent of the neutrino energy, since the explicit dependence on E of b is compensated by the fact that at resonance $N_e \sim E^{-1}$.

To compute the ν conversion probability $P_{\nu_e \nu_e}$, eq. (8), the knowledge of P and of $c2\theta_m$ at the neutrino production point is required. P obviously only depends on the resonance value of b , while

$$c2\theta_m = \frac{c2\theta - a}{\sqrt{(c2\theta - a)^2 + (b + s2\theta)^2}} \quad (17)$$

is sensitive to the value of b only if the production point is near the resonance. From the previous discussion it is clear then that in the case $s2\theta < |b_r|$, the role of $s2\theta$ in the neutrino oscillations is played by b_r . Hence, to have interesting oscillation effects as in the ordinary MSW, for which $s2\theta \gtrsim 10^{-2}$ was required, we need

$$|G^e + G^p + Y_n G^n|_{e\alpha} \gtrsim 10^{-2} \frac{G_F}{2}. \quad (18)$$

We also expect a similar general behaviour of the solar ν_e fluxes as a function of the neutrino squared mass difference Δ . For $\Delta \simeq 10^{-4} \text{ eV}^2$ there should be an important adiabatic conversion of ν flavors for $b_r \gtrsim 10^{-2}$, while for $\Delta \times b_r \simeq 10^{-7.5} \text{ eV}^2$ there should be a non-adiabatic regime.

IV – One interesting model leading to non-standard neutrino properties such as masses or flavor changing neutrino interactions, is the well known supersymmetric extension of the standard model ⁸ including some R -parity violating interactions⁹. In particular, we will concentrate in the following lepton number violating contributions to the superpotential

$$\lambda_{ijk} L_i L_j E_k^c \quad (a) \quad , \quad \lambda'_{ijk} L_i Q_j D_k^c \quad (b), \quad (19)$$

where L , Q , E^c , D^c are the usual lepton and quark $SU(2)$ doublets and singlets respectively and i, j, k are generation indices. Due to the contraction of the $SU(2)$ indices in eq. (19-a), the λ_{ijk} should be antisymmetric under the exchange of i and j . In the following, we will take these Yukawas to be real so that they do not violate CP.

These couplings induce a finite neutrino Majorana mass matrix through one-loop diagrams involving a lepton (quark) and a slepton (squark) line. For instance, the leptonic loop leads, in the usual supersymmetric model with soft breaking terms as arising from low energy supergravity, to^{10,11}

$$\delta m_{ij} \simeq \frac{\lambda_{ikk'} \lambda_{jk'k}}{8\pi^2} m_k m_{k'} \frac{\tilde{m}}{m_0^2}, \quad (20)$$

where \tilde{m} is a typical supersymmetric mass (of $O(100 \text{ GeV})$ if supersymmetry is to solve the naturalness problem), m_i is the lepton mass and all sleptons were assumed almost degenerate at a mass m_0 .

For the MSW solution to the solar neutrino problem, with small vacuum mixing angles and discarding fine-tuned cancellations between the ν masses, we need the diagonal entry $\delta m_{\alpha\alpha}$ to be $\sim 10^{-2} \div 10^{-4} \text{ eV}$ with the remaining entries in the ν mass matrix much smaller. This can be easily fulfilled by many possible choices of reasonably small λ (or λ') constants. Several phenomenological constraints typically impose¹² $\lambda, \lambda' \lesssim 10^{-1}$, although some stronger bounds apply to the products of constants that are involved in rare processes such as $\mu \rightarrow e\gamma$, $\mu \rightarrow ee\bar{e}$, etc.¹⁰. Clearly, it is also possible to generate in this way the mixing angles required in the ordinary MSW, but we want here to concentrate on the resonant conversion induced by the flavor-changing neutrino interactions previously discussed.

Turning now to the neutrino interactions with the medium, we will analyze first the interactions with the electrons. The superpotential (19-a) leads to the lagrangian

$$\mathcal{L} = \lambda_{ijk} [\bar{\nu}_L^i \bar{e}_R^k e_L^j + \bar{e}_L^j \bar{e}_R^k \nu_L^i + \bar{e}_R^{k*} (\bar{\nu}_L^i)^c e_L^j - i \leftrightarrow j] + h.c. \quad (21)$$

In the case of ν_e scattering off electrons, this leads to an effective low-energy interaction of the form

$$\mathcal{L}_{eff} = -\frac{\lambda_{1j1} \lambda_{lj1}}{m_{\tilde{e}_L^j}^2} \bar{e}_R \gamma_\mu e_R \bar{\nu}_L^l \gamma^\mu \nu_L^e. \quad (22)$$

The scattering through \tilde{e}_R exchange is not present because it vanishes in the s-channel ($\lambda_{11k} = 0$) while for \tilde{e}_R exchange in the t-channel ν_e only scatters off e^+ . Hence,

$$\sqrt{2} G_{1l}^e = -\frac{\lambda_{1j1} \lambda_{lj1}}{2m_{\tilde{e}_L^j}^2} \quad (23)$$

and consequently the condition for resonant oscillations to take place in the sun, eq. (18), now reads

$$\lambda_{1j1} \lambda_{lj1} \gtrsim 1.5 \times 10^{-3} \left(\frac{m_o}{100 \text{ GeV}} \right)^2. \quad (24)$$

If $l = 2$ ($\nu_e \leftrightarrow \nu_\mu$ conversion), the interaction is mediated by $\tilde{\tau}_L$ exchange ($j = 3$). However, the same product of couplings $\lambda_{131} \lambda_{231}$ appears in the radiative decay $\mu \rightarrow e\gamma$ and hence¹⁰ it is severely constrained to be less than $10^{-5}(m_o/100 \text{ GeV})^2$. The allowed values of $G_{e\mu}^e$ are then too small. The remaining possibility is to have $l = 3$ ($\nu_e \leftrightarrow \nu_\tau$ conversion), mediated by $\tilde{\mu}_L$ exchange. The product $\lambda_{121} \lambda_{321}$ is bounded by the products of the individual bounds obtained in ref. 12, i.e. $\lambda_{121} \lambda_{321} \lesssim 0.04 \times 0.09(m_o/100 \text{ GeV})^2$ (the bound from the process $\tau \rightarrow e\gamma$ is weaker). This just marginally allows parameter values for which there can be interesting oscillation effects¹³. Note also that these products of couplings are not involved in the elements of the neutrino mass matrix (see eq. (20)) and so they are not further constrained.

For the $\nu - q$ interactions, the superpotential (19-b) leads to

$$\mathcal{L} = \lambda'_{ijk} [\tilde{d}_L^j \tilde{d}_R^k \nu_L^i + (\tilde{d}_R^k)^* (\tilde{\nu}_L^i)^c d_L^j] + h.c. \quad (25)$$

(omitting terms without neutrinos). This results in a low energy effective lagrangian

$$\mathcal{L}_{eff} = \frac{\lambda'_{ijk} \lambda'_{lmk}}{2m_{\tilde{d}_R}^2} [\tilde{\nu}_L^l \gamma_\mu \nu_L^i \tilde{d}_L^m \gamma^\mu d_L^j] - \frac{\lambda'_{ijk} \lambda'_{ljn}}{2m_{\tilde{d}_L}^2} [\tilde{\nu}_L^l \gamma_\mu \nu_L^i \tilde{d}_R^k \gamma^\mu d_R^n]. \quad (26)$$

Since the neutrinos only interact with the down quarks in the nucleons through the exchange of down type squarks, the constraint (18) translates into

$$(1 + 2Y_n) G_{e\alpha}^d \gtrsim 10^{-2} \frac{G_F}{2}, \quad (27)$$

so that the couplings involved should satisfy $|\lambda' \lambda'| \gtrsim 10^{-3}(m_{\tilde{q}}/100 \text{ GeV})^2$. In the case of $\nu_e \nu_\mu$ conversion, scattering through \tilde{s}_L exchange involves the product of couplings $\lambda'_{121} \lambda'_{221}$, while the scattering through \tilde{b}_L exchange involves $\lambda'_{131} \lambda'_{231}$. However, since these couplings also induce the process $\mu \rightarrow e\gamma$, they are very suppressed and $\nu_e \nu_\mu$ conversion is not allowed. Instead, it is possible to generate $\nu_e \nu_\tau$ conversion by exchange of either left or right down-type squarks, since there are no strong bounds on λ_{3jk} alone, while the bound from $\tau \rightarrow e\gamma$ is¹⁴ $\lambda'_{1jk} \lambda'_{3jk} \lesssim 5 \times 10^{-2}(m_{\tilde{q}}/100 \text{ GeV})^2$. It is interesting to note that for this model the required couplings could be probed at a τ factory¹⁴.

V— In conclusion, in the same way as a small neutrino mixing in vacuum can be amplified producing significant oscillations of the neutrinos that cross a resonance layer while propagating in a medium, we have shown that similar effects can be obtained in the presence of flavor-changing neutrino interactions. This has important applications to solar neutrinos since allowed strengths for those new interactions can lead to a solution to the solar neutrino deficit even for negligibly small vacuum mixings.

We have shown how lepton number violating couplings that can be present in the minimal supersymmetric extension of the standard model are able to generate the required flavor non-diagonal neutrino interactions with quarks and leptons, as well as the necessary neutrino masses, taking into account the experimental bounds on the new couplings.

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